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Effect of micro-randomness on macroscopic properties and fracture of laminates

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Abstract Composite materials demonstrate a considerable extent of heterogeneity. A non-uniform spatial distribution of reinforcement results in variations of local properties of fibrous laminates. This non-uniformity not only affects effective properties of composite materials but is also a crucial factor in initiation and development of damage and fracture processes that are also spatially non-uniform. Such randomness in microstructure and in failure evolution is responsible for non-uniform distributions of stresses in composite specimens even under externally uniform loading, resulting, for instance, in a random distribution of matrix cracks in cross-ply laminates. The paper deals with statistical features of a distribution of carbon fibres in a transversal cross-sectional area in a unidirectional composite with epoxy matrix, based on various approaches used to quantify its microscopic randomness. A random character of the fibres' distribution results in fluctuations of local elastic moduli in composites, the bounds of which depend on the characteristic length scale. A lattice model to study damage and fracture evolution in laminates, linking randomness of microstructure with macroscopic properties, is discussed. An example of simulations of matrix cracking in a carbon fibre/epoxy cross-ply laminate is given.

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Introduction

Manufacturing fibrous composites inevitably leads to some heterogeneity of the obtained materials. Their microstructural analysis is usually concerned with manufacture-induced defects such as fibre/matrix debonding, interface cracks, microvoids, fibre ruptures and kinks, etc. According to some estimates [1], in any cross-sectional area one tenth of fibres are broken in the manufacturing process. These defects not only result in deterioration of material's properties but also serve as stress concentrators and, hence, nuclei of macroscopic fracture initiation. Still, even neglecting their effect, another source of randomness is important for many types of laminates, namely, a non-uniform distribution of their constituents. In uniaxial fibrous composites the distribution of defects along the fibres and the variation of their volume fraction are assumed to be two main factors determining stochasticity in properties of these composites [2]. The image analysis technique, applied to cross-sectional areas of T300/914 specimens, demonstrated a high level of variability: for a composite with an average volume fraction of fibres 55.9% the minimum and maximum level of the observed volume fractions were 15% and 85%, respectively [2]. Such a high scatter undermines any use of schemes, based on ideal periodic or quasi-periodic arrangement of fibres, to estimate the effective properties and, especially, failure parameters of such composites. Only in the case of the effective longitudinal stiffness (in the direction parallel to the axes of aligned fibres), the effect of randomness can be neglected for a relatively large size of a transversal window.

Various attempts were undertaken to quantify random distributions of fibres and to account for their effect. Some elements of classification for point patterns, formed by centroids of fibres, were introduced in [3]. There, a second-order intensity function, defining a number of points expected to be situated within a given distance, was suggested as a most suitable parameter to characterise random sets of fibres in unidirectional composites.

An account of the random distribution of fibres in unidirectional composites is implemented mainly numerically, in terms of representative volume elements (RVE) with periodic boundary conditions, with RVEs incorporating the details of microstructure. Here, either real microscopic images of fibres within some window, directly discretised into finite elements (see, e.g. [4]), or generated, statistically equivalent microstructures are used. Various techniques were suggested to obtain a statistically equivalent ensemble of fibres (or, more generally, reinforcement). The so-called random sequential absorption is used in [5] to reproduce distributions of fibres in a unidirectional composite for an arbitrary volume fraction in the interval form zero to that for closest pack triangular structure. Another approach employs a variable-box Monte Carlo technique to describe the microstructure of the glass/epoxy unidirectional composite [6]. An alternative way to study the effect of the volume fraction of fibres on effective properties of composites with randomly distributed fibres is to use a so-called multi-particle effective field method [7].

Though finite element simulations of RVEs allow a detailed study of microstructural aspects of deformation processes [8] within a single unit cell, the problem of transferability of the results, obtained for a relatively small window (normally, with dimensions 100–500 μm), to a composite component/structure is a non-trivial matter. Besides, the character of randomness in local distributions of fibres depends also on the characteristic length size. To overcome these deficiencies of a single-RVE approach, this paper presents a lattice model that represent a specimen as a set of elements, each with its own effective properties, linked to statistics of the volume fraction of fibres for a respective scale. The discussion of this model is preceded by a quantitative study of the effect of the window size on the randomness in the distribution of fibres in a unidirectional carbon fibre–epoxy composite.

Analysis of microstructure

Statistical approaches

In unidirectional fibrous composites axial variations in the volume fraction of fibres due to their misalignment are considerably smaller than those in transversal cross sections. Hence, a micrograph of a transversal cross-sectional area of a unidirectional carbon/epoxy composite, containing a sufficiently large number of fibres, is analysed to estimate the extent of non-uniformity in the spatial distribution of constituents. The size of the studied window is 345 μm × 250 μm, i.e. its dimensions are larger than the thickness of a single standard ply in cross-ply laminates (normally, 120–150 μm); it contains (centroids of) 603 fibres with diameter $d^f = 10$ μm.

The analysis is apparently limited to the study of statistics of fibres since epoxy matrix is an embedding media. To characterise the spatial randomness of this set of fibres, various approaches and parameters are applied. The first step is to analyse a mutual position of the nearest neighbours of fibres in the set (Fig. 1). The graph vividly demonstrates non-uniformity in distribution of fibres, which becomes more obvious if we also consider histograms of spacings between fibres (Fig. 2) and orientation of couples of nearest neighbours (Fig. 3). Here, the spacing between two fibres means the distance between their centroids diminished by d^f. It is apparent that though the majority of fibres have the spacing less than 1 μm, still a considerable minority (35%) are separated by a distance larger than that, in some cases—up to five times.

In terms of orientation, the set of fibres demonstrates its random character (Fig. 2). But still it could not be considered as fully random, i.e. isotropic in the transverse plane (this would correspond to a dashed straight line in Fig. 2): the maximum density of distribution—for a 10° band—is 2.6 times higher than the minimum one. Hence, though there is no obvious anisotropy in the transverse distribution of carbon fibres in this unidirectional composite, it is not fully transversally isotropic at this scale.

Two other approaches can be used to characterise the spatial distribution of fibres. The first one is based on the second-order intensity function [3, 9] that describes an increase in the average number of fibres (their centroids)

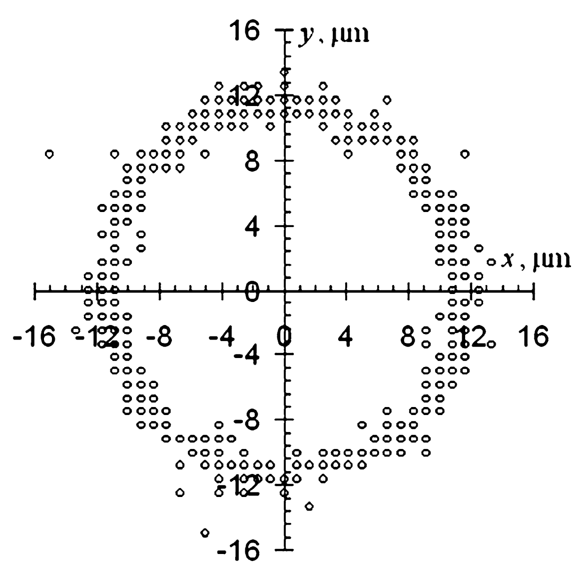

Fig. 1 Position of the nearest neighbours in distribution of fibres in transversal cross-section

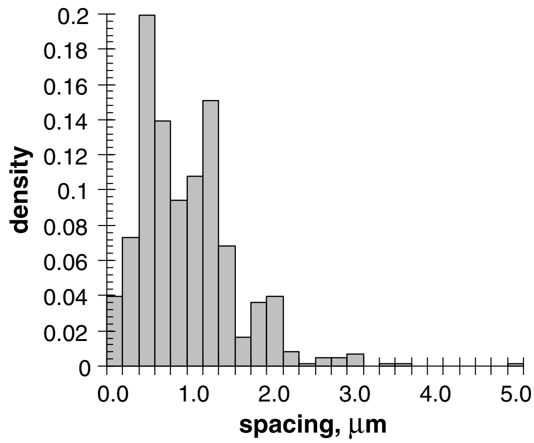

Fig. 2 Distribution of spacing between the nearest neighbours in the set of carbon fibres transversal cross-section

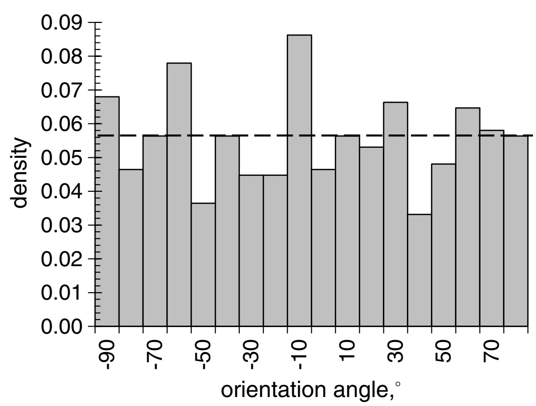

Fig. 3 Orientational distribution of carbon fibres (pairs of nearest neighbours) in transversal cross-section

with the distance r from an arbitrary fibre (its centroid). This intensity function for the studied area is given in Fig. 4. Another approach is based on the so-called radial distribution function that can be derived from the second-order intensity function [9–11]. The radial distribution function characterises not only short-range random properties of the set, as in a study of the pairs of nearest neighbours, but also its middle- and large-range parameters. Its graph (Fig. 5) has a typical structure, observed for other fibrous composites (see, e.g. [11]): a high peak, followed by some fluctuations, and an obvious asymptote of unity for large values of r, linked with the average spatial density of fibres. The fully random distributions of points in the plane should have their radial distributions functions equal to unity for all values of r. The first peak is linked to the finiteness of the fibres radius that mean a transition from random statistics of points to so called *hard-core random distributions* since no fibre centroid can be situated closer to the centroid of any fibre that d^f.

The next step is to study the effect of the window size on the type of local randomness in the spatial distribution of fibres. To implement this, the analysed area of the composite's transversal cross section is discretised into cells with dimensions changing from one discretisation to another, and the volume fraction of fibres is determined for each cell. This approach is close to the idea of a *mesoscale window* that was suggested and applied to random composites in [12, 13] to study the scale dependence of their effective moduli. Let us notice that in contrast to the approaches used above, here we deal not with the distributions of centroids of fibres but with an actual part of the cross-sectional area occupied by fibres. In this sense, the window size can be smaller than the fibre diameter d^f.

Histograms of the volume fraction of fibres v^f for the same 25 bands, width of each 0.04, are given in Fig. 6 for various window sizes. The general trend with the decrease in the window size is obvious: the distribution width increases with respective flattening of histograms. Two bounds for distributions of the volume fraction of fibres can be introduced—maximal (v^f_{max}) and minimal (v^f_{min}). Their graphs for the varying window size demonstrate two natural trends (Fig. 7). Firstly, for sufficiently small length scales, these two bounds tend to respective mono-phase asymptotes: $v^f_{max} \to 1$ and $v^f_{min} \to 0$ (i.e. the volume

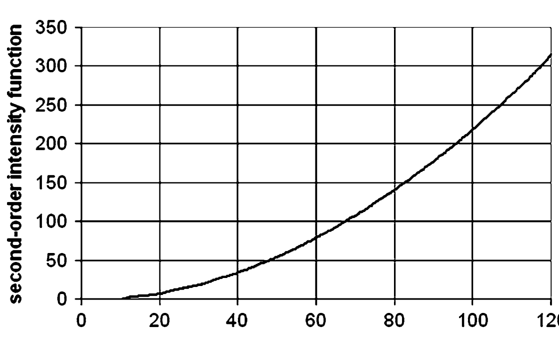

Fig. 4 Second-order intensity function for a distribution of fibres

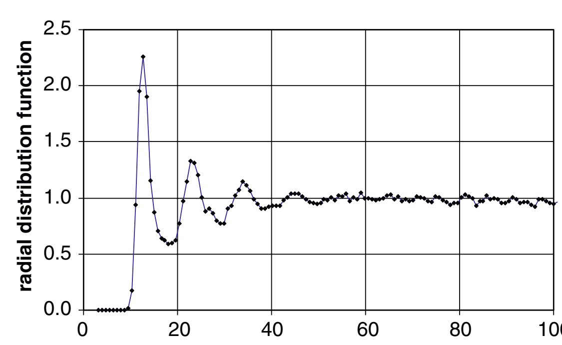

Fig. 5 Radial distribution function for a cross section of unidirectional fibrous composite

fraction of the matrix $v^m \rightarrow 1$). Obviously, the former limit occurs at the length scale below d^f due to different shapes of the window and reinforcement's cross-section. The latter limit is attained at the higher length scale, even larger than d^f, for the average volume fraction of fibres $\langle v^f \rangle = 0.55$ of the studied carbon/epoxy composite. Secondly, both bounds should converge at high levels of the length scale to the average value:

$$\left. \begin{array}{l} v^f_{min} \\ v^f_{max} \end{array} \right\} \rightarrow \langle v^f \rangle.$$

This trend is also distinct (Fig. 7) but the full convergence of the bounds is not reached even at the length scale of 115 μm.

Multifractal characterisation

The statistical analysis above has vividly demonstrated the effect of the length scale on the type of randomness for distributions of fibres in the unidirectional carbon/epoxy composite. To quantify the spatial scaling of non-uniform distributions, a multifractal formalism [14, 15] can be used. For a stochastic distribution in some area, the local

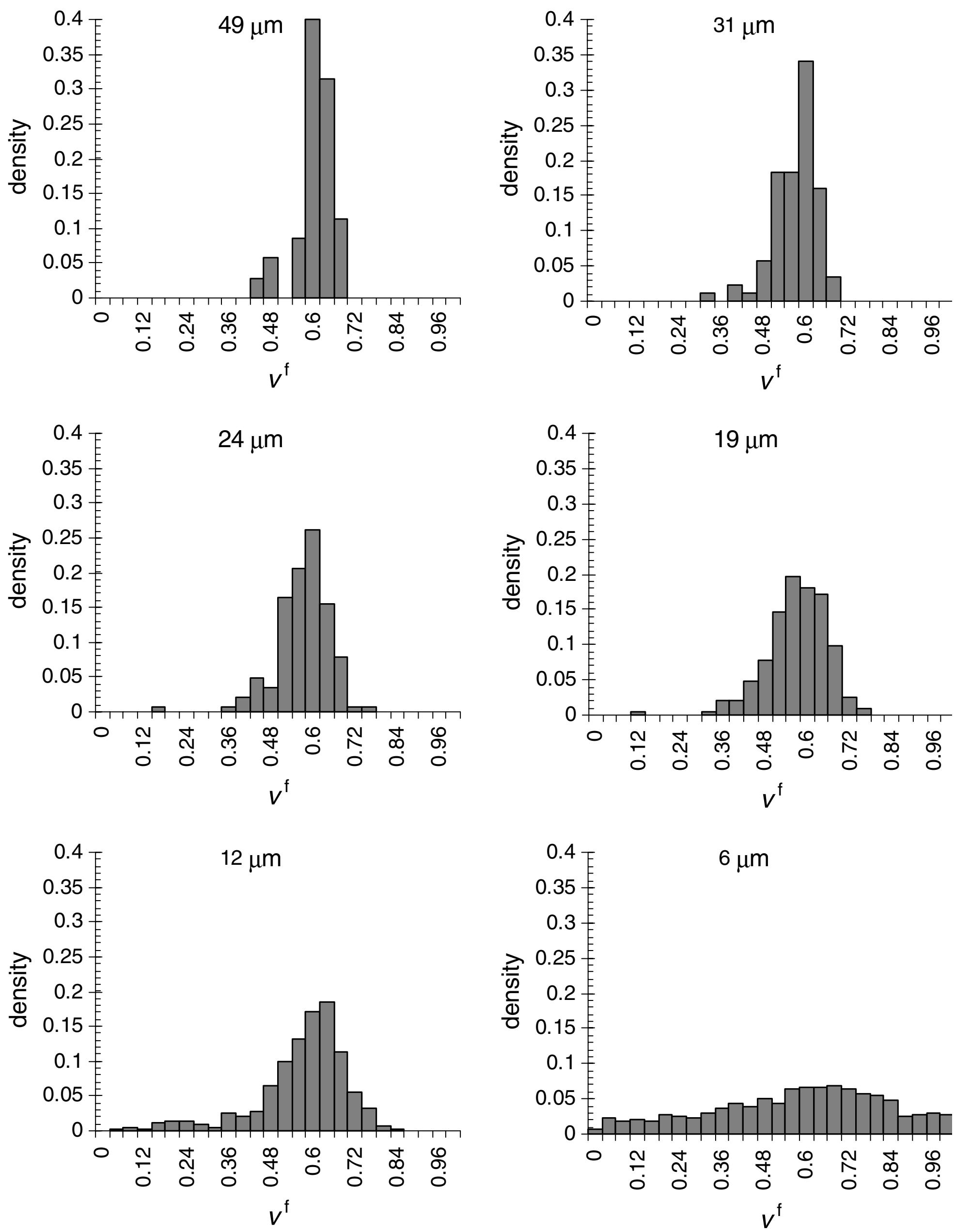

Fig. 6 Effect of the length scale (window size) on distribution of the volume fraction of fibres

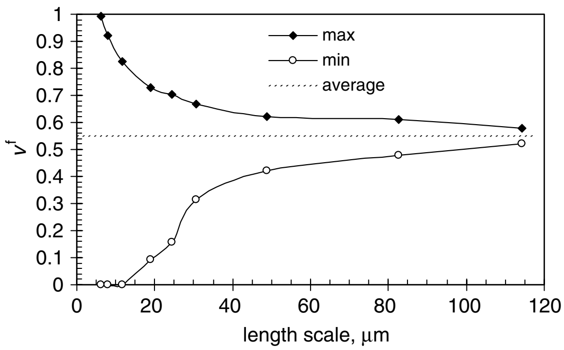

Fig. 7 Effect of the length scale on bounds of distributions of the volume fraction of fibres

probability P_i introduced for a set of elements (boxes), compactly covering this area and labelled by index i, scales as:

$$P_i(l) \propto l^{\alpha_i}, \tag{1}$$

where l is a length scale (box size) and α_i is a respective exponent (known also as singularity strength). The number of elements with probability characterised by the same exponent is linked to the length scale by the fractal (Hausdorff) dimension $f(\alpha)$:

$$N(\alpha) \propto l^{-f(\alpha)}. \tag{2}$$

Hence, $f(\alpha)$ describes the entire (finite) spectrum of scaling exponents for non-uniform distributions. A direct application of Eqs. (1) and (2) results in inaccurate estimates for $f(\alpha)$ due to poor convergence [16], so another procedure was suggested in [15, 17] based on parametric presentations for f and α. A modification of this approach for the case of a 2D distribution of fibres in a transverse cross section has the following form:

$$\alpha = \lim_{l\to 0} \frac{1}{\log l} \sum_{i=1}^{N_x} \sum_{j=1}^{N_z} \mu_{ij} \log n_{ij}, \tag{3}$$

$$f = \lim_{l\to 0} \frac{1}{\log l} \sum_{i=1}^{N_x} \sum_{j=1}^{N_z} \mu_{ij} \log \mu_{ij}, \tag{4}$$

where $\mu_{ij} = n_{ij}^q \left(\sum_{k=1}^{N_x} \sum_{m=1}^{N_z} n_{km}^q \right)^{-1}$.

Here l is a length scale (box or window size), $M = N_x N_z = L_x L_z / l^2$ is a total number of boxes necessary to cover the entire area $L_x \times L_z$ under study, $n_{ij} = N_{ij}/N_{\mathrm{f}}$ is a relative number (probability) of (centroids of) fibres within the box (i, j), N_{f} is the total number of fibres in the analysed area.

The calculated multifractal spectrum $f(\alpha)$ for the carbon fibre/epoxy unidirectional composite is presented in Fig. 8. Apparently, the non-uniform distribution of fibres in the area under study has a multifractal character since the calculated graph demonstrates the properties of a multifractal spectrum:

(1) It is a cup convex which lies under the bisector $f = \alpha$.
(2) It has a single connection point with this bisector where $f'(\alpha) = 1$. The value of $f(\alpha)$ in this point (for $q = 1$) is called the informational dimension [18, 19].
(3) The maximum value of $f(\alpha)$ curve is the box-counting dimension D of the geometric support of the measure: $D = 2$ for a distribution over a 2D region.

The width of the multifractal spectrum is linked to the extent of randomness of the distribution: with the increase in uniformity of the distribution the spectrum tends to the point $f(D) = D$, which is characteristic for total isotropy.

Effective properties and random fracture

The type of randomness in the spatial distribution of fibres, analysed in the previous section, affects both the effective properties of composites and evolution of failure in them. Still, a majority of mechanical approaches uses the average volume fraction of fibres $\langle v^{\mathrm{f}} \rangle$ as a single global parameter in simulations of the behaviour of composite components and structures. But local fluctuations in the level of v^{f} lead to non-uniformity of spatial fields of the Young's moduli. As a result, various parts of the same composite will be exposed to different levels of stresses even under the macroscopically uniform boundary conditions (in forces and/or displacements). For instance, the effective axial modulus of the composite $\bar{E}_{11}$ can be presented as

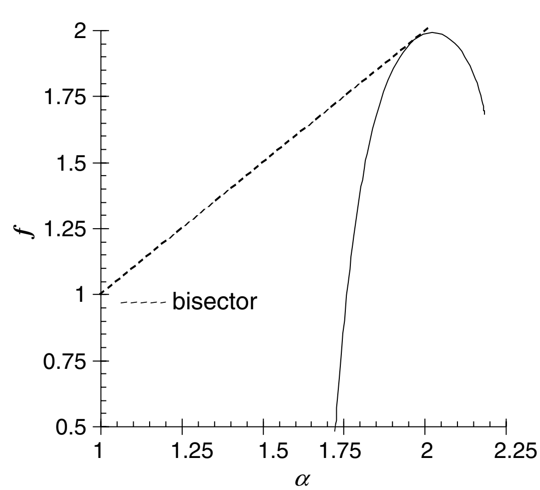

Fig. 8 Multifractal spectrum $f(\alpha)$ for the distribution of carbon fibres in transversal cross-section

$$\bar{E}_{11} = E_{11}^{\mathrm{RoM}} + \Delta E_{11}, \tag{5}$$

where E_{11}^{RoM} is the axial modulus calculated according to the linear rule of mixtures, and ΔE_{11} is a non-linear term that is negligibly small compared to E_{11}^{RoM} for a unidirectional carbon/epoxy composite. After rearranging the standard presentation of the linear rule of mixtures to the form

$$E_{11}^{\mathrm{RoM}} = v^{\mathrm{f}}\left(E_{\mathrm{L}}^{\mathrm{f}} - E^{\mathrm{m}}\right) + E^{\mathrm{m}}, \tag{6}$$

proportionality of the local magnitude of the elasticity modulus to the local volume fraction of fibres becomes obvious. In Eq. (6) $E_{\mathrm{L}}^{\mathrm{f}}$ is the longitudinal module of fibres (transversely isotropic in the case of carbon ones) and E^{m} is the Young's modulus of the (isotropic) matrix.

A relatively slow convergence of $v_{\max}^{\mathrm{f}}$ and $v_{\min}^{\mathrm{f}}$ to the average volume fraction of fibres $\langle v^{\mathrm{f}} \rangle$ with the increase in the length scale (see Fig. 7) has severe implications for numerical modelling of laminates. For instance, application of 2D finite elements presupposes the use of at least several nods along the transverse (through-thickness) direction of a lamina. For a single ply's thickness 120–150 μm, the maximum element's dimension should not exceed 30–50 μm. Hence, to reproduce adequately spatial (non-uniaxial) stress distributions in such composites (especially with a purpose to determine possible places for crack generation) a model should account for a scatter in their local properties.

A statistical analysis of the microscopic structure of the carbon/epoxy composite can be used to estimate respective bounds for the effective elastic moduli based on the scatter in the volume fraction of fibres at the length scales, relevant for macroscopic numerical modelling. Various schemes can be used for this purpose for carbon/fibre laminates, e.g., the self-consistent approach [20, 21], Mori–Tanaka method [22, 23], concentric cylinder assemblage model [24–26], or method of cells [27]. The last three approaches give very close results for many fibre-reinforced laminates [28].

Table 1 demonstrates ratios of the maximum values of the local effective elastic moduli to the minimum ones obtained using the concentric cylinder assemblage model for the studied case of the distribution of carbon fibres (here $\bar{E}_{22}$ and $\bar{G}_{12}$ are the effective transverse modulus and effective axial shear modulus, respectively). This estimation demonstrates that the maximum values of the effective axial moduli (both normal and shear) are 50% higher than the minimum ones for the length scale of 50 μm. For the length scale 30 μm these ratios exceed 2. The scatter in the effective transverse modulus is lower, but still its maximum local values are 20%–40% higher than the minimum ones.

Table 1 Ratios of local effective elastic moduli of carbon-fibre composite for various length scales

Length scale	$\bar{E}_{11}^{\max}/\bar{E}_{11}^{\min}$	$\bar{E}_{22}^{\max}/\bar{E}_{22}^{\min}$	$\bar{G}_{12}^{\max}/\bar{G}_{12}^{\min}$
30 μm	2.09	1.41	2.06
50 μm	1.47	1.21	1.53

Still, this randomness can play a significant part in evolution of stresses and damage in fibrous composites as was shown in [8, 9]. It is important not only for unidirectional composites but also for cross-ply laminates exposed to uniaxial loading. In this case an internal layer, formed by 90° plies, is loaded transversely and the major damage mechanism at the initial stages of loading (either quasi-static or tensile fatigue) is matrix cracking (see, e.g. [29]).

Experimental studies of matrix cracking in cross-ply laminates under fatigue conditions manifest the stochastic nature of the matrix-cracking process under fatigue [30–32]: distances between two neighbouring matrix cracks show a considerable—up to hundreds of per cent—scatter. Various schemes and measures can be introduced to quantify this randomness. The Weibull distribution function could be used to characterise the set of inter-crack distances for laminates at different stages of their loading history [32, 33]. In [34] histograms for numbers of matrix cracks in bands of a constant width, into which the test specimen is divided, were used. The multi-fractal formalism can also be used to characterise the type of randomness in a matrix-crack set [35–37].

Comprehension of the stochastic character of matrix cracking was reflected in different models, employing various schemes to incorporate the material's randomness. Among the suggested approaches are the introduction of the initial distribution of microcracks (flaws) [38], the use of spatial strength distributions [31, 39], randomness in the specific surface energy [40] and a more general phenomenological scheme based on the suggestion of the Itô stochastic differential equation for fatigue-damage accumulation within the concept of continuum damage mechanics [41]. These schemes were used in a combination with fracture mechanics or stress transfer rules (shear-lag analysis). Analytical approaches can also be successfully used to predict effective properties of laminates with non-uniform crack distributions [42]. Stress distributions in laminates for specific statistical realisations of crack sets were analysed using either quasi-unidirectional shear stress analysis [43] or 2D finite-element simulations [33].

To estimate the effect of randomness in microstructure (i.e., in a spatial distribution of fibres in a transversal cross section) of a $[0_n/90_m/0_n]$ laminate, a multi-scale lattice model is used. Details of these model are discussed elsewhere [36, 44], so its main features are only briefly considered below. In contrast to schemes based on the analysis of a representative volume element, here the entire specimen (i.e. its longitudinal, through-thickness cross section) is presented as a 2D lattice of non-overlapping elements with properties varying from one element to another due to differences in their microstructure. The spatial non-uniformity in transverse stiffness of the 90° layer is considered as the main source of the initial randomness in material's properties, affecting the matrix cracking process. To adequately incorporate this feature into the model, histograms of the volume fraction of fibres, obtained from the microstructural analysis for the respective scale length that coincides with the elements' dimensions, are used to determine the type of randomness in material's local stiffness. The three-parameter Weibull distribution with the cumulative distribution function is used to describe this data:

$$F(\bar{C}) = 1 - \exp\left[-\left(\frac{\bar{C}-\gamma}{\eta}\right)^{\beta}\right], \quad (7)$$

where $\bar{C} = E_{90}^{ij}/E_{90}^{\min}$ is the ratio of the local transverse modulus to its minimum value for the respective length scale (elements' dimensions). The obtained values of three parameters of this distribution are β = 13.0, η = 0.395 and γ = 0.76. The graphical presentation of the function is given in Fig. 9 together with the microstructure-based data.

The randomness in stiffness results in the non-uniform distribution of longitudinal (in 0° direction) stresses even under conditions of the uniform external load. The initial local level (i.e., for a laminate without matrix cracks) of this stress for elements can be calculated using the stress-renormalizing coefficients [36, 44]. They reflect two factors affecting the local stress level: (a) the global transfer of the external stress linked to the stacking order of the laminate and (b) the local variations in the transverse modulus.

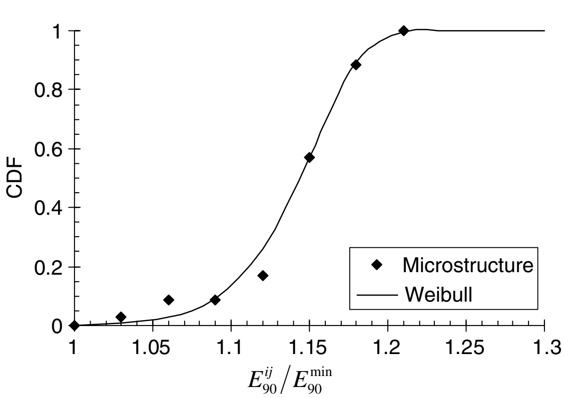

Fig. 9 Cumulative distribution functions for relative transversal modulus: analytical (Weibull) and microstructural data

The initial random stress causes spatially non-uniform evolution of ensembles of microscopic defects. This process can be described in terms of continuum damage mechanics by means of introduction of the damage parameter at the macroscopic level of the model. This parameter reflects the deformational effect of evolution of the ensemble of microscopic defects. Due to orientational degeneracy of transverse cracks in relatively thin 90° layers under an axial load, a scalar damage parameter can be introduced with the damage evolution law for tensile fatigue in the form of a modified Coffin–Manson relation [44].

The damage parameter describes the macroscopic effect of disperse damage; hence an additional condition is necessary to determine generation of a macroscopic matrix crack as a result of localisation of damage growth. In the suggested model a transition from the disperse damage accumulation to a local event of a macroscopic fracture is linked to overcoming of the threshold damage value. Attainment of this threshold in any element is considered to be a moment of initiation of the transverse crack in a respective part of the specimen. It is assumed that this crack instantly crosses the entire thickness of the 90° layer; this is confirmed by experimental observations (e.g., [29]). Obviously, due to spatially non-uniform damage evolution linked with randomness in stress levels, some of the elements, which demonstrate higher damage accumulation rates, will attain the critical damage level earlier than others.

Generation of the first (and subsequent) matrix crack changes the stress field in the direct vicinity of the crack due to formation of its stress-free surfaces—it is known as a *shielding effect*. The effect of shielding on the axial stress can be treated in different ways, with analytical schemes being mainly based on variants of the shear-lag approach [43, 45, 46]. To incorporate the shielding effect into the suggested lattice model, an additional multiplier is introduced into stress-renormalizing coefficients, which is obtained by integration of the known analytical relations for the elements of the lattice model. Calculations show that generation of the matrix crack disturbs the stress distribution in $[0_2/90_8/0_2]$ laminates as far as 2 mm from it. With the magnitude of inter-crack spacing at the developed stages of fatigue being considerably less than 1 mm, on overlap of shielding zones from two neighbouring transverse cracks significantly change the axial stress along the entire spacing, and even the next-nearest neighbouring cracks have a non-vanishing effect on the stress magnitude. Hence, the model employs the superposition principle, accounting for interacting shielding zones from multiple matrix cracks.

In the case of laminates with a considerably thick 90° layers, with matrix cracks growing through their thickness during several loading cycles, matrix cracks disturb the distribution of axial stresses not only in the longitudinal direction but also in the transverse one. For instance, the matrix crack that occupies not the entire thickness causes considerable stress concentration near its tip. This could be accounted by introduction of local stress-concentration coefficients [44, 47] based on the expansion of fracture mechanics to the case of lattice models. In such a case, the interplay of two mechanisms—the initial spatial randomness and stress concentration—are responsible for the direction of crack propagation. Under conditions of the uniaxial tensile fatigue, the latter mechanism attempts to straighten the crack in the direction, normal to that of loading. Still, the material's randomness could be responsible for local crack deviations from the transverse direction in cases when elements in adjoining columns have already accumulated a significant amount of damage.

In general terms, the suggested algorithm, which is based on the local damage evolution relation, can be considered as a mapping of a dynamic matrix of stress-renormalizing coefficients onto the lattice of elements, covering the 90° layer. Such a matrix incorporates effects of the initial microstructural randomness as well as the disperse evolution of damage and its transition to spatially localised matrix cracking. This scheme could also be expanded to include other failure mechanisms such as delamination, provided that either analytical or numerical estimates of their effects on stress distributions are available. There are obvious limits to the suggested approach: in a case of a complicated type of loading determination of a matrix of renormalizing stress coefficients will be infeasible. A principle advantage of the suggested scheme in comparison with the finite-element method is its straightforward analysis of multiple cracking. In finite-element schemes any newly formed transverse crack (or its incremental growth in the case of thick 90° layers) should cause the reformulation of the boundary-value problem due to the formation of new traction-free surfaces.

An example of implementation of the suggested lattice scheme is the study of matrix cracking in a specimen of the $[0_2/90_8/0_2]$ carbon/epoxy laminate. Figure 10 demonstrates a distribution of matrix cracks in a specimen of T300-934 (axial length 50 mm) loaded by tensile fatigue with the maximum cyclic stress 450 MPa at various stages of the loading history. It is obvious that initial stages of matrix cracking are characterised by a considerable extent of randomness and a large scatter in the spacing length. With a transition to an advanced fatigue stage, newly formed transverse cracks tend to form closer to the mid-spacing. Apparently, this is due to the mutual action of the shielding zones from neighbouring cracks, with their length at this stage exceeding the average spacing. Still, at all stages there are cracks that are formed relatively close to existing ones (see Fig. 10b, d). In this case, the effect of a large local fluctuation in material's properties is so strong that even the local stress reduction due to the shielding effect cannot prevent cracking.

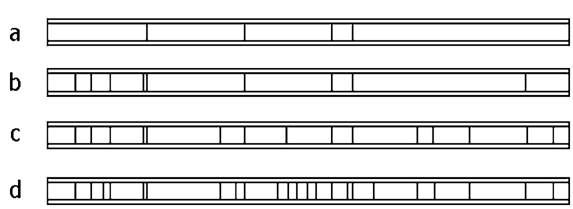

Fig. 10 Positions of matrix cracks in $[0_2/90_8/0_2]$ laminate at different moments of loading history: (**a**) 100 cycles, (**b**) 4×10^3 cycles, (**c**) 10^5 cycles and (**d**) 2×10^5 cycles. Vertical to horizontal scale 2:1

The obtained results regarding the number and position of matrix cracks in a studied specimen should be understood in a probabilistic sense: the change in the initial spatial distribution of the material's properties will result in another set of matrix cracks for the same loading conditions and history. Hence, any result is a single statistical realisation. Still, the multifractal analysis demonstrates similarity of these statistical realisations (i.e. sets of transverse cracks in specimens of the same structure) due to the closeness of their multifractal spectra for similar loading conditions.

Conclusions

Microstructural studies of carbon/epoxy laminate demonstrate a non-uniform character of distributions of fibres in transversal cross sections. Such non-uniformity can considerably affect evolution of deformation and failure processes in composites. Several parameters are employed to characterise both the short-range randomness in the direct vicinity of a fibre in terms of nearest-neighbour statistics and medium- to long-range non-uniformity, using the second-order singularity function and radial distribution function. All the schemes show that the type of randomness depends on the respective length scale (window size). The fibres' distribution also demonstrates some non-trivial—multifractal—scaling. The fluctuating level of the volume fraction of fibres results in the spatial non-uniformity of the material's local elastic parameters.

These microscopic studies, justifying and quantifying the material's stochasticity, have serious implications for numerical modelling of laminates: any attempt to adequately reproduce spatial (non-uniaxial) stress distributions in such composites should account for a scatter in their local properties. This is becoming even more important for modelling of the damage and failure evolution, linked to a transition from the disperse accumulation of microscopic defects to localization of the fracture process in the form of cracks and delamination zones.

A lattice model that describes not a single representative volume element but a macroscopic specimen, discretised into elements with different properties, is used to implement such approaches. It allows a detailed analysis into the effect of the composite's randomness on its effective properties and on development of matrix cracking under tensile fatigue. The model bridges macro and micro levels of description: microscopic processes are accounted for in each element (as in a case of RVEs) in terms of the damage parameter, while the macroscopic processes of stress redistribution—due to the spatial randomness in the material's properties, structure of composites as well as matrix cracking—are considered at the scale of the entire specimen (lattice of elements). The set of matrix cracks, obtained in simulations, is shown to be equivalent (both in terms of statistics and scaling) to experimentally observed sets of matrix cracks in carbon/epoxy laminates.

Interaction of microscopic randomness in distribution of fibres with macroscopic redistributions of stresses due to the increase in the number of transverse cracks under tensile fatigue determines the character of matrix cracking: from more random at its initial stage with relatively large inter-crack spacing to more ordered at the advance stage. The main ordering factor is the shielding effect causing considerable reduction in longitudinal stress near cracks in the 90° layer. With initial spacing considerably exceeding zones of this effect, matrix cracking is governed mainly by the spatial randomness in material's properties. Increasing the crack density results in the growth of the portion of the specimen's length occupied by shielding zones, limiting the probable nucleation sites for new matrix cracks mainly to parts of the specimen situated around the middles of spacing between the neighbouring cracks.

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